Parameter and uncertainty identification for a multidimensional population balance model for granulation

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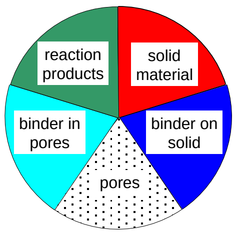
A methodology for the estimation of model parameters and their uncertainties for a multivariate population balance model for granulation is presented. The outcome of the detailed population balance model is locally approximated by responses surfaces in order to allow quick computation in the parameter estimation approach, using three different objective functions.

1. Background and idea

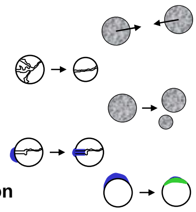
The evolution of the particles during a wet granulation process is described by a multidimensional population balance framework with following particle model and incorporated transformations [1]:

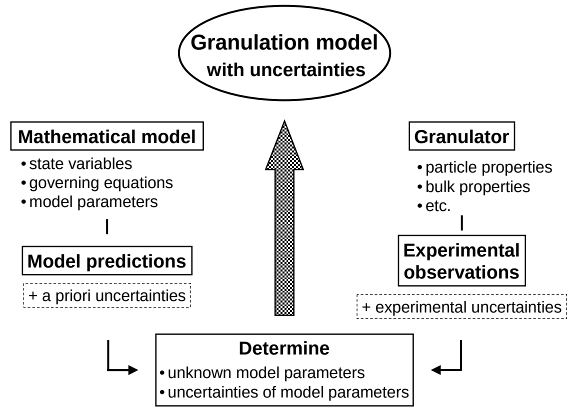
Particle model



Transformations

- **Coalescence**
- **Compaction**
- **Breakage**
- **Penetration**
- **Chemical reaction**





2. Theory

Choice of "universal" parameters

- **model response with parameters** x

$\eta = \eta(x)$ **with** $x = (x_1, \ldots, x_K)$

- **experimental datum**

$\eta^{\exp} = \eta_0^{\exp} \pm \sigma^{\exp}$

- **each parameter** x **can be represented by**

$x = x_0 + c\xi \qquad \xi \sim N(0,1)$

> For a simple linear model, K=1:
>
> $\eta(x) = A + Bx \qquad \eta(x,c,\xi) = A + B(x_0 + c\xi)$
>
> $\mu(x_0) = E[\eta(x_0,c,\xi)] = A + Bx_0 \qquad \sigma(c) = \sqrt{\mathrm{Var}(\eta(x_0,c,\xi))} = \sqrt{B^2c^2}$

- **optimal values for *unknown parameters*** x_0^* **and associated uncertainties** c^*

$$(x_0^*, c^*) = \underset{x_0,c}{\mathrm{argmin}}[\Phi(x_0,c)]$$

- **with objective function (moment matching)**

$$\Phi(x_0,c) = \sum_{i=1}^{N}\left([\eta_{0,i}^{\exp} - \mu_i(x_0,c)]^2 + [\sigma_i^{\exp} - \sigma_i(x_0,c)]^2\right)$$

i = experiment index ($i = 1,\ldots,N$)
μ, σ = model response
$\eta_0^{\exp}, \sigma^{\exp}$ = experimental values

- **and constraints**

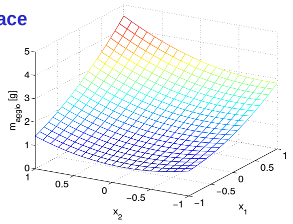
$$x_{0,k,\mathrm{low}} \le x_{0,k} \le x_{0,k,\mathrm{up}} \quad (k = 1,\ldots,K)$$

$$0 \le c \le c^{(0)}$$

Where do we get model response $\eta(x)$ and $\mu(x_0,c)$, $\sigma(x_0,c)$ respectively from?

– directly from model: numerically expensive
– local approximation – **response surface**
– e.g. 2nd order response surface

$$\eta(x) = \beta_0 + \sum_{k=1}^{K}\beta_k x_k + \sum_{k=1}^{K}\sum_{l\ge k}^{K}\beta_{kl}x_kx_l$$



Alternative objective function

- **expected least squares**

$$\Phi(x_0,c) = \sum_{i=1}^{N}\left([\eta_{0,i}^{\exp} - \mu_i(x_0,c)]^2 + [\sigma_i^{\exp} - \sigma_i(x_0,c)]^2\right) - \sum_{i=1}^{N}2\sigma_i^{\exp}\left(\beta_{i,i}c_i + \sum_{k=1}^{i}\beta_{ki,i}x_{0,k}c_i + \sum_{l\ge i}^{K}\beta_{il,i}x_{0,l}c_i\right)$$

- **weighted expected least squares: experimental uncertainties as weights [2]**

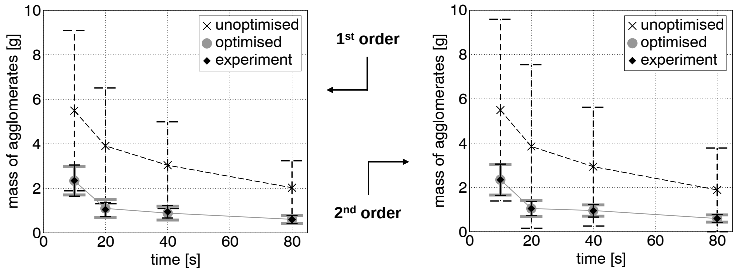
3. Results

Experimental system

- **wet granulation of sugar pareils with aqueous PEG4000 (50%) solution in food mixer operated with impeller speed of 900 and 1200 rpm [3]**
- **determination of mass of agglomerates at 4 different process times**

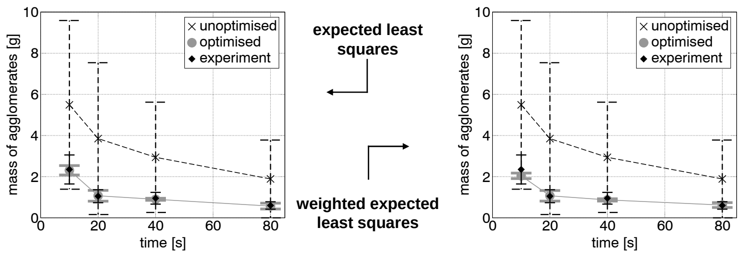
1st order vs. 2nd order approximation

- **moment matching objective function (impeller speed of 900 rpm)**



Different objective functions

- **2nd order response surfaces (impeller speed of 900 rpm)**



4. Conclusions

- **estimation of unknown model parameters and associated uncertainties for multivariate population balance model**
- **choice of moment-matching objective function recommend as number of experimental points is independent of number of unknown parameters**
- **use of methodology for falsification of models in future work**

References

[1] A. Braumann, M. Kraft, M. Mort, *Powder Tech.* **2010**, 197*(3)*, 196
[2] A. Braumann, P. L. W. Man, M. Kraft, *Ind. Eng. Chem. Res.* **2010**, *49(1)*, 428
[3] T. Simmons, R. Turton, P. Mort, *Fifth World Congress on Particle Technology* **2006**

Acknowledgements

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